

**Exercise 41**

Find the limit.

$$\lim_{t \rightarrow 0} \frac{\tan 6t}{\sin 2t}$$

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**Solution**

Rewrite the limit in terms of one that is known.

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{\tan 6t}{\sin 2t} &= \lim_{t \rightarrow 0} \frac{\sin 6t}{\cos 6t \sin 2t} = 3 \lim_{t \rightarrow 0} \left( \frac{1}{\cos 6t} \cdot \frac{\sin 6t}{6t} \cdot \frac{2t}{\sin 2t} \right) \\ &= 3 \left( \lim_{t \rightarrow 0} \frac{1}{\cos 6t} \right) \left( \lim_{t \rightarrow 0} \frac{\sin 6t}{6t} \right) \left( \lim_{t \rightarrow 0} \frac{2t}{\sin 2t} \right) \\ &= 3 \left( \lim_{t \rightarrow 0} \frac{1}{\cos 6t} \right) \left( \lim_{t \rightarrow 0} \frac{\sin 6t}{6t} \right) \left( \lim_{t \rightarrow 0} \frac{1}{\frac{\sin 2t}{2t}} \right) \\ &= 3 \left( \lim_{t \rightarrow 0} \frac{1}{\cos 6t} \right) \left( \lim_{\alpha \rightarrow 0} \frac{\sin \alpha}{\alpha} \right) \left( \frac{1}{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}} \right) \\ &= 3(1)(1) \left[ \frac{1}{(1)} \right] \\ &= 3 \end{aligned}$$